TABLE I
RESULTS USING FOUR-SLUG TUNER CIRCUIT

Number of Diodes	Diode Voltage (V)	Diode Current (A)	Peak Power Output (W)	Efficiency (%)	Frequency (MHz)
1	180	1.35	78	32.1	496
2	210	2.8	170	29.0	488
3	220	4.2	180	19.5	510
4	210	6.2	300	23.0	510

It should be noted that only two FD-300 diodes out of approximately 12 tested produced TRAPATT oscillations with efficiency as high as 38 percent. Both of these diodes had identical breakdown voltages of 223 V and both had a substantially higher voltage drop than the other diodes tested when operating in the TRAPATT mode. Several other diodes produced TRAPATT oscillations in this circuit but with slightly less efficiency and in some cases greater noise.

REFERENCES

[1] R. J. Chaffin and E. P. EerNisse, "A poor man's TRAPATT oscillator," *Proc. IEEE* (Lett.), vol. 58, pp. 173–174, Jan. 1970.
[2] D. F. Kostishack, "UHF avalanche diode oscillator providing 400 watts peak power and 75 percent efficiency," *Proc. IEEE* (Lett.), vol. 58, pp. 1282–1283, Aug. 1970.
[3] R. J. Chaffin, "High-power TRAPATT oscillations from parallel-connected low-cost diodes," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-18, pp. 985–986, Nov. 1970.
[4] H. J. Prager, K. K. N. Chang, and S. Weisbrod, "High-power, high-efficiency silicon avalanche diodes at ultra high frequencies," *Proc. IEEE* (Lett.), vol. 55, pp. 586–587, Apr. 1967.

Application of Deschamps' Graphical Method to Measurements of the Scattering Coefficients of Multiport Waveguide Junctions

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Abstract—**A method for measuring scattering coefficients of a multiport waveguide junction is presented. It is based on the application of Deschamps' graphical technique to reduced multiport junctions, and analysis of measurements done with either matched or nearly matched loads terminated at all but the input and output ports. Averaging and least-square fitting are introduced to reduce errors from measurements.**

In an earlier correspondence, Stein [1] presented an extension of Deschamps' graphical method [2] to measurements of scattering coefficients of multiport waveguide junctions. In that method, reflection coefficients are measured at the input port with all other ports terminated by movable shorts. By changing the positions of the movable shorts systematically, a series of Deschamps' graphs may be constructed. Scattering coefficients may then be calculated from the results obtained in these graphs. Since this method requires a large number of measurements with still larger numbers of constructions of Deschamps' graphs, it becomes impractical for junctions with more than four ports. In the present correspondence, we report a direct application of Deschamps' graphical method to a multiport waveguide junction, which is terminated with matched or nearly matched loads at all but the input and output ports. This enables one to reduce the required VSWR measurements and the Deschamps-type constructions drastically. A brief description of our method is presented in the following paragraphs.

Manuscript received April 7, 1971; revised May 27, 1971. This work was supported by the U. S. Atomic Energy Commission.
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Without losing generality, let the input port be port one and the output port be port two. One arrives at the following determinant equation from the definition of scattering matrix.[1]

$$\begin{vmatrix} S_{11}-\dfrac{1}{\Gamma_1} & S_{12} & S_{13} & \cdots & S_{1n} \\ S_{12} & S_{22}-\dfrac{1}{\Gamma_2} & S_{23} & \cdots & S_{2n} \\ S_{13} & S_{23} & S_{33}-\dfrac{1}{\Gamma_3} & \cdots & S_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ S_{1n} & S_{2n} & S_{3n} & \cdots & S_{nn}-\dfrac{1}{\Gamma_n} \end{vmatrix} \equiv 0 \qquad (1)$$

where $\Gamma_j = a_j/b_j$, and a_j and b_j are complex numbers representing incoming and outgoing waves to the junction. Using standard expansion rules for determinant and assuming that the mismatches from port three through port n are small, one neglects some of the higher order terms of Γ_j with $j \leq 3$, and arrives at

$$\frac{1}{\Gamma_1} = S_{11}' + \frac{S_{12}'^2\Gamma_2}{1 - S_{22}'\Gamma_2} \qquad (2)$$

where

$$S_{ii}' = S_{ii} + \sum_{j=3}^{n} \frac{S_{ij}^2\Gamma_j}{1 - S_{jj}\Gamma_j}, \qquad \text{with } i = 1 \text{ or } 2 \qquad (3)$$

$$S_{12}' = S_{12} + \sum_{j=3}^{n} \frac{S_{1j}S_{2j}\Gamma_j}{1 - S_{jj}\Gamma_j}. \qquad (4)$$

A comparison shows that (1) is in the same form as [2, eq. (11)], where the scattering coefficients S_{11}, S_{22}, and S_{12} of the two-port junction are replaced, respectively, by modified scattering coefficients S_{11}', S_{22}', and S_{12}' of the reduced multiport. One can then obtain these modified scattering coefficients by adopting Deschamps' method with input and/or output connected at different ports for different sets of measurements. With perfectly matched loads, $\Gamma_j = 0$ for $j = 3$ to n. The multiport scattering coefficients are directly measured.

In the case of slight mismatches at port three through port n, S_{11} may be solved from measurements done according to the following procedure.

1) Measuring the reflection coefficient Γ_1 at an input port with nearly matched loads at all other ports. The reflection coefficient can be obtained from (2) with higher order terms of Γ_j, $j = 2$ through n neglected,

$$\frac{1}{\Gamma_1} = S_{11} + \sum_{j=2}^{n} \frac{S_{1j}^2\Gamma_j}{1 - S_{jj}\Gamma_j} \equiv S_{11}^{(0)}. \qquad (5)$$

2) Replacing the load at port k with a movable short with the other ports remaining unchanged. By using Deschamps' measuring process, S_{11}', S_{kk}', and S_{1k}' can again be measured. Let this S_{11}' be $S_{11}^{(k)}$, then

$$S_{11}^{(k)} \equiv S_{11} + {\sum_{j=2}^{n}}' \frac{S_{1j}^2\Gamma_j}{1 - S_{jj}\Gamma_j} \qquad (6)$$

where $\sum'$ signifies summation over j, except for the $j = k$ term. The

[1] It should be noted that a reciprocal junction is implied in this definition and also that the characteristic impedances of all arms are assumed to be equal. However, the method can be easily extended to apply to the nonreciprocal junctions and junctions with unequal characteristic impedance on different arms.

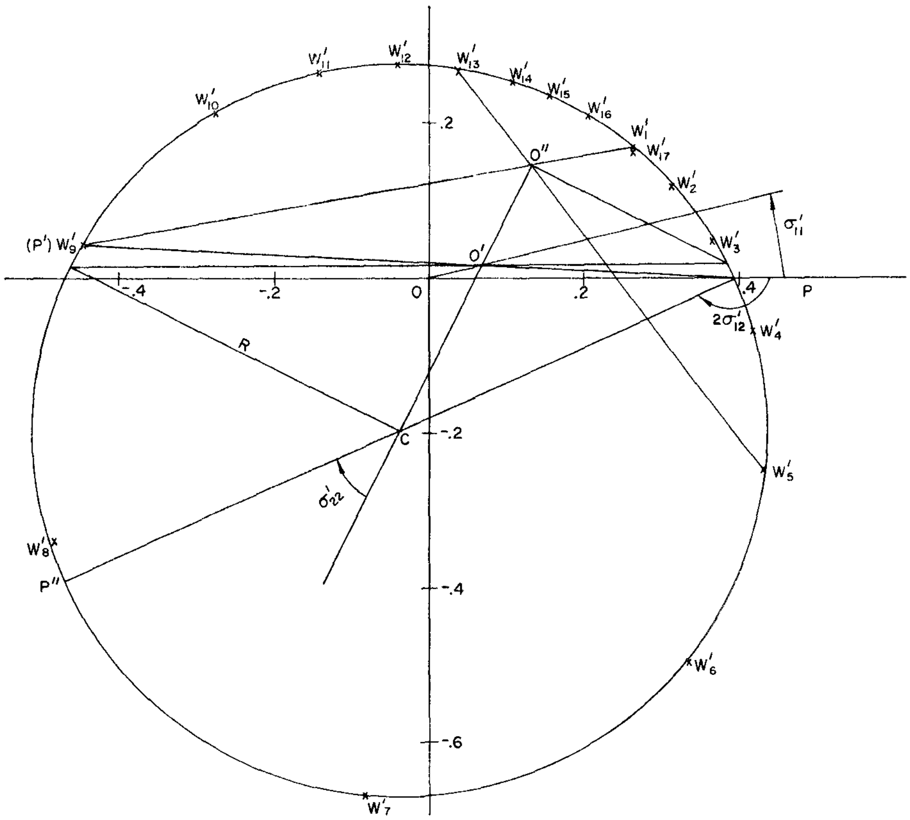

Fig. 1. Deschamps' construction for a reduced multiport waveguide junction with measurement done at port one and movable short placed at port two, and the other ports terminated with nearly matched loads. σ_{11}', σ_{22}', and σ_{12}' are phases of S_{11}', S_{22}', and S_{12}', respectively. Deschamps' notations are adopted in this graph.

difference between (5) and (6) yields the correction for mismatch:

$$\Delta_{1k} \equiv \frac{S_{1k}^2\Gamma_k}{1 - S_{kk}\Gamma_k} = S_{11}^{(0)} - S_{11}^{(k)}. \tag{7}$$

With these corrections, the multiport scattering coefficients may be readily obtained from (3) and (4).

Scattering coefficients of an L-band hybrid-tee waveguide junction were measured at 1.271 GHz to verify our procedure. By neglecting mismatches, 6 Deschamps' constructions (as those shown in Fig. 1) are required for the entire scattering matrix. With corrections, 9 Deschamps' constructions and 3 additional measurements outlined in 1) are needed. The microwave source was carefully matched. VSWRs of the loads were 1.03 or less. Standing-wave measurements were done at input arm with a movable short placed at $n\lambda/32$ from the output reference plane of the hybrid tee, where λ is the wavelength and $n=0, 1, \cdots, 16$. The 17th point was provided mainly as a check point. The reflection coefficient was obtained from averaging the amplitudes and phases of several slotted-line measurements. These reflection coefficients W_i' were then least square fitted to a circle to find its radius and center (R and point C in Fig. 1). The inconocenter O' was obtained from O'', which is the average of all intersections of chords $W_i'W_{i+8}'$, where $i=1, 2, \cdots, 8$. The modified scattering coefficients were directly calculated using (6) and (7). The whole process was done by IBM 360 and lasted several seconds. The correction Δ_{1k} in the above measurements are small and all $|S_{ij}|$ (reflection coefficient measured at arm i with movable short at arm j) differ from $|S_{ji}|$ by less than one percent.

References

[1] S. Stein, "Graphical analysis of measurements on multi-port waveguide junctions," *Proc. IRE* (Corresp.), vol. 42, p. 599, Mar. 1954.

[2] G. A. Deschamps, *J. Appl. Phys.* vol. 24, p. 1046, 1953.

Mode Chart for E-Plane Circulators

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***Abstract*—The mode chart of the E-plane junction circulator is given. The geometry considered consists of two ferrite disks placed against the narrow walls at the plane of symmetry of a symmetrical 3-port E-plane waveguide junction.**

It is experimentally found to exhibit two modes. One of these modes has a resonant frequency which increases with the spacing between the two ferrite disks. The other mode has a resonant frequency which decreases with the spacing between the disks. Both modes are independent of the disk spacing when the spacing is large. It is also found that the frequency of both modes is proportional to the thickness of the ferrite disks. Finally, circulators obtained by magnetizing each of the two modes circulate in opposite directions.

Experimental results on a circulator obtained in this way are included.

A number of authors have made brief references in the literature to the E-plane junction circulator [1]–[7]. Such references have been mostly of an experimental nature. One advantage of the E-plane junction compared to the more usual H-plane one is its peak power capability. The purpose of this correspondence is to give the mode chart for this type of circulator. Mode charts of circulators may be constructed by observing that their center frequency coincides with the frequency at which the VSWR of the reciprocal 3-port junction

Manuscript received April 12, 1971; revised July 6, 1971.
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